# The Fiscal Eigenvalue

*When Sovereign Debt Stops Converging*

Primary Forcing Refinancing Memory and the Washington Tokyo Control Problem

> *A buyback can alter the surface of a market. Only a reaction rule can alter the law of motion beneath it.*

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# Abstract

The Arithmetic of Escape argued that a Treasury buyback can treat market structure while leaving the fiscal law of motion untouched. This technical companion states that law exactly and follows its implications further. Debt dynamics contain a forcing term, a persistence coefficient, and a refinancing kernel. The primary deficit is the forcing term. The ratio (1 + r) / (1 + g) is the one-period persistence coefficient - the fiscal eigenvalue of the scalar linearisation. The maturity structure determines how rapidly a change in market yields enters the effective rate. A sovereign can therefore have an eigenvalue below one and still converge toward an intolerably high debt ratio when the primary deficit remains positive; as the growth-interest gap approaches zero, that stationary ratio becomes unbounded.

The companion also corrects the calibration used in Paper III. CBO directly reports a 3.4 percent average rate on debt held by the public in 2026. Applying that series, rather than the earlier rounded ratio, to debt at 101 percent of GDP and a primary deficit of 2.6 percent produces an indicative stabilisation requirement of approximately 1.5 to 2.5 percent of GDP under nominal-growth assumptions of 4.5 to 3.5 percent. On the rounded CBO aggregates used here, that is roughly $500 billion to $820 billion per year. The correction does not alter the published paper's direction. It increases the scale of the structural adjustment required.

A buyback is then decomposed into four separate objects: cash flow, market liquidity, public duration, and fiscal solvency. The September 10 operation accepted approximately $5.2 billion against a $6 billion maximum while long yields rose amid a broader inflation and oil shock. That observation is consistent with the scale argument, but it is not a controlled experiment and is not treated as proof. The paper then develops a coupled U.S.-Japan control model. Higher Japanese rates can lower the cost of hedging U.S. Treasuries into yen while simultaneously raising the domestic JGB alternative and Japan's own debt-service path. The sign of the effect on Treasury demand is therefore theoretically ambiguous and must be measured. Central-bank liquidity swaps are assigned to dollar-funding stress, not to a persistent exchange-rate target.

The proposed governance result is the Sovereign Convergence Rule. It requires authorities to publish the fiscal eigenvalue, the convergence residual, and the refinancing-memory coefficient as one state vector; to assign each policy target to its proper instrument; and to legislate a state-contingent primary-balance reaction strong enough to offset adverse debt feedback. The paper is an engineering framework, not a recommendation about the political distribution of adjustment.


### Analytical Map

| Object | Symbol | Question answered | Critical condition |
|---|---|---|---|
| Fiscal eigenvalue | $\lambda$ | How much of last period's debt ratio survives into this period | $\lambda < 1$ is necessary for scalar convergence |
| Convergence residual | $\Gamma$ | Is the debt ratio rising after all current forces are combined | $\Gamma \leq 0$ stabilises or reduces debt |
| Refinancing memory | $q$ | How rapidly do marginal yields enter the effective interest rate | Lower $q$ slows pass-through but may carry term cost |
| Closed-loop spectral radius | $\rho$ | Does the coupled U.S.-Japan policy system return toward its target | $\rho < 1$ in the local linear model |

# I Sequence and Advance

Paper III remains the published record. A mature doctrine should neither rewrite that record nor defend a rounded input after a better series is identified. This document is therefore registered as Advanced Technical Companion III A. It sits beside The Arithmetic of Escape and leaves the announced Paper IV unchanged. Its purpose is not to make the earlier prose longer. Its purpose is to expose the hidden state variables that the first paper compressed into one sentence about feedback.

The advance is threefold. First, the debt equation is written without the approximation that removes the growth denominator. Second, the effective interest rate is modelled as a distributed lag created by the maturity profile. Third, Washington and Tokyo are treated as controllers acting on a shared market rather than as one actor and one source of demand. Those three changes move the doctrine from magnitude comparison to system design.

The central claim is deliberately narrower than a forecast. A liquidity operation cannot, by itself, satisfy a solvency condition. A maturity operation cannot, by itself, create a primary surplus. A policy-rate decision made for domestic inflation cannot be assumed to produce one sign in a foreign bond market. Once those categories are separated, credible coordination becomes an assignment problem: name the state, name the target, name the actuator, and publish the condition under which the actuator is released.

# II The Exact Law of Motion

Let D denote nominal debt held by the public at the end of a period, Y nominal GDP, r the effective nominal interest rate applied to the inherited debt stock, g nominal GDP growth, P the primary deficit stated as a positive number, and F stock-flow adjustments not recorded in the primary deficit. Dividing the nominal budget identity by current-period GDP gives the exact one-period debt law.

$$d_t = [(1 + r_t) / (1 + g_t)]\, d_{t-1} + b_t + s_t$$

*Equation 1 Exact debt-ratio law*

Here d = D / Y, b = P / Y, and s = F / Y. The change in the debt ratio follows immediately.

$$\Delta d_t = [(r_t - g_t) / (1 + g_t)]\, d_{t-1} + b_t + s_t$$

*Equation 2 Exact debt drift*

The common approximation drops the denominator and writes Δd approximately equal to (r - g)d + b. That approximation is useful for intuition. It is not needed here. The exact equation reveals two distinct mechanisms that should never be collapsed. The primary deficit b is a forcing term: it adds to the ratio each period. The coefficient multiplying inherited debt is a propagation term: it determines how strongly the existing stock persists.

### The Fiscal Eigenvalue

$$\lambda_t = (1 + r_t) / (1 + g_t)$$

*Equation 3 Fiscal eigenvalue of the scalar linearisation*

The term eigenvalue is used precisely and locally. In the one-state linear equation with constant parameters, λ is the coefficient governing the propagation of a deviation in inherited debt. If λ is below one, a deviation decays in the absence of new forcing. If λ equals one, the inherited ratio does not decay. If λ exceeds one, the inherited ratio expands even before a primary deficit is added. In a time-varying or multi-state system, the relevant object is no longer one scalar; it is the dominant eigenvalue, or spectral radius, of the local transition matrix developed later in this paper.

The phrase debt stabilisation is often used as though $r < g$ were sufficient. It is not. The condition $\lambda < 1$ concerns the propagation of inherited debt. A positive primary deficit can still push the system toward a high stationary ratio. Holding b, r, g, and s constant, the stationary debt ratio exists only when $g > r$ and is given by Equation 4.

$$d^* = (b + s)(1 + g) / (g - r)$$

*Equation 4 Stationary debt ratio when g exceeds r*

The denominator is the danger. As g approaches r from above, d* becomes arbitrarily large. At r equal to or above g, a positive forcing term admits no finite stable positive fixed point. This is the deeper meaning of the arithmetic of escape: it is possible for the system to be technically convergent and institutionally unacceptable at the same time.

### The Convergence Residual

$$\Gamma_t = b_t + s_t + [(r_t - g_t) / (1 + g_t)]\, d_{t-1}$$

*Equation 5 One-period convergence residual*

The residual Γ is the operational test. A value above zero means the ratio is still rising under the current policy and macroeconomic state. A value of zero stabilises the ratio for that period. A value below zero reduces it. The stabilising primary deficit, stated as a positive deficit, is therefore the maximum deficit consistent with Γ equal to zero.

$$b^{stab}{}_t = [(g_t - r_t) / (1 + g_t)]\, d_{t-1} - s_t$$

*Equation 6 Stabilising primary deficit*

A sustainable primary deficit is not a moral entitlement and a primary surplus is not a universal requirement. The sign and scale depend on growth, the effective rate, the inherited debt ratio, and stock-flow adjustments. The equation is an identity. The political choice lies in how the required balance is reached and how risk is shared.

## III Calibration and Correction

CBO's February 2026 baseline reports a fiscal-year deficit of $1.9 trillion, or 5.8 percent of GDP; debt held by the public at 101 percent of GDP; a primary deficit of 2.6 percent of GDP; and net interest at 3.3 percent of GDP. It projects debt at 120 percent of GDP and net interest at 4.6 percent of GDP by 2036. CBO also directly estimates the average interest rate on debt held by the public at 3.4 percent in 2026, rising to 3.9 percent in the final years of the projection. [1]

### Calibration Note

Paper III described the current effective rate as approximately 3.0 percent by dividing a rounded $1 trillion interest bill by a rounded $33 trillion debt stock. That was an insufficiently precise choice because CBO publishes the direct average-rate series. This companion replaces the rounded quotient with CBO's 3.4 percent estimate. It also uses the exact growth denominator. The consequence is shown below.

| Nominal growth g | Fiscal eigenvalue λ | Sustainable primary deficit | Required adjustment | Approx annual amount | Frozen-parameter d* |
|---|---|---|---|---|---|
| 4.5% | 0.9895 | 1.06% of GDP | 1.54% of GDP | $500bn | 247% of GDP |
| 4.0% | 0.9942 | 0.58% of GDP | 2.02% of GDP | $660bn | 451% of GDP |
| 3.5% | 0.9990 | 0.10% of GDP | 2.50% of GDP | $820bn | 2,691% of GDP |

Method: d = 101 percent, r = 3.4 percent, b = 2.6 percent, s = zero. Dollar amounts apply each percentage-point adjustment to approximately $32.8 trillion of GDP implied by CBO's rounded $1.9 trillion and 5.8 percent figures. Because those inputs are rounded, the dollar amounts are presented to the nearest $10 billion. The stationary ratios are not forecasts. They freeze all parameters indefinitely to expose sensitivity to the small denominator g - r.

The correction changes the lower and upper bounds from the earlier approximate range of $400 billion to $700 billion to approximately $500 billion to $820 billion. It does not prove that any particular package would lower long yields, and it does not select taxes against spending. It does establish a scale condition: a measure that cannot plausibly move the primary balance by this order of magnitude should not be described as the fiscal solution.

The table also shows why a binary debate over whether r is below g is inadequate. At 3.5 percent nominal growth, λ remains fractionally below one, yet the frozen-parameter stationary ratio exceeds twenty-six times GDP because the 0.1 percentage-point growth advantage is too small to absorb a persistent 2.6 percent primary deficit. A sovereign can stand on the convergent side of the line while being economically nowhere near an acceptable destination.

### Scale and Timing

A one-percentage-point permanent increase in the effective rate applied to the full 101 percent debt ratio would eventually add approximately 1.01 percent of GDP to annual gross interest expense before offsets, about $330 billion on the same GDP base. The word eventually is indispensable. Existing securities do not all reprice at once. The maturity structure determines when the shock arrives. A $6 billion buyback maximum and a $330 billion steady-state annual interest sensitivity are therefore not like-for-like flows. Their comparison is a scale diagnostic, not a cash-accounting identity.

## IV Refinancing Memory

The effective rate is not today's ten-year or thirty-year yield. It is the weighted coupon and discount cost embedded in the outstanding portfolio, adjusted by the rate on newly issued debt and the timing of redemptions. CBO explicitly models the maturity structure when projecting the average rate. [1] A compact renewal approximation makes the mechanism visible.

$$\bar{r}_{t+1} = (1 - q_t)\,\bar{r}_t + q_t\,y_t + a_t$$

*Equation 7  Refinancing-memory approximation*

The bar over r denotes the portfolio effective rate, y the all-in marginal yield on the debt refinanced or newly issued during the period, q the share of the relevant stock repriced during that period, and a valuation, inflation-indexation, fee, and composition adjustments. In the simplest constant-q model, the weight of an old rate observation decays geometrically at (1 - q) to the power of the number of periods. A higher q means shorter memory and faster pass-through. A lower q means longer memory and slower pass-through.

$$\text{Half life} \approx \ln(0.5) \;/\; \ln(1 - q)$$

*Equation 8  Memory half life under constant renewal*

This is why the financing profile belongs inside the fiscal model rather than in a separate debt-management appendix. A sovereign with a high bill share, large near-term maturities, floating-rate exposure, or inflation-linked principal has a larger effective q. The same market-rate shock then reaches the budget faster. A longer nominal fixed-rate portfolio lowers q, but usually requires paying the term premium. Maturity extension is therefore insurance against refinancing variance. It is not guaranteed to be cheaper ex post.

### Refinancing at Risk

A single weighted-average-maturity number hides concentration. The more useful public measure is refinancing at risk over a stated horizon H. It should include principal maturing within H, projected net borrowing within H, floating-rate resets, and the rate-sensitive component of inflation-linked obligations.

$$\mathrm{RaR}^{H} = [\mathrm{M}^{H} + \mathrm{N}^{H} + \mathrm{FR}^{H} + \mathrm{IL}^{H}] \;/\; \mathrm{Y}$$

*Equation 9  Refinancing at risk over horizon H*

RaR does not claim that every maturing dollar is dangerous. It measures the portion of the liability structure exposed to a new price before the horizon closes. Publishing RaR at one, three, and five years would make the fiscal consequences of maturity choices legible. A buyback that retires long fixed-rate debt and is financed with bills can improve off-the-run liquidity while raising short-horizon RaR. Both statements can be true.

## V The Algebra of a Buyback

A buyback should be evaluated across four ledgers. The first is cash. The second is market liquidity. The third is duration transferred between the sovereign and the public. The fourth is the fiscal law of motion. Confusion begins when an improvement in one ledger is reported as an improvement in all four.

| Ledger | What can change | What does not follow automatically | Primary evidence |
|---|---|---|---|
| Cash | Timing of redemptions and financing needs | A permanent fall in debt if the purchase is funded by new issuance | Financing statement and net borrowing |
| Liquidity | Off-the-run scarcity, dealer balance-sheet use, price dispersion | A lower equilibrium term premium | Bid-ask spreads, fails, depth, relative-value dispersion |
| Duration | Interest-rate risk held by the public and renewal risk held by Treasury | Lower decade-wide funding cost | DV01 retired versus DV01 reissued |
| Fiscal dynamics | Only transaction costs, discounts, or accompanying primary measures | A change in the primary forcing term | Γ before and after the complete package |

Let B be the market value of securities retired and R the market value of replacement issuance attributable to the operation. Ignoring small transaction and accounting effects, a cash-financed purchase initially reduces liquid balances and a debt-financed purchase replaces one liability with another. The approximate change in the debt ratio is governed by net financing, not by the gross headline B.

$$\Delta \mathrm{d} \text{ from operation} \approx (\mathrm{R} - \mathrm{B} + \text{costs} + \text{stock flow effects}) \;/\; \mathrm{Y}$$

*Equation 10  Buyback cash identity*

Duration transfer can be written in dollar-value-of-a-basis-point terms. Let DV01B denote the interest-rate sensitivity retired and DV01R the sensitivity reissued.

$$\Delta \mathrm{DV01} \text{ held by public} \approx \mathrm{DV01}^{R} - \mathrm{DV01}^{B}$$

*Equation 11  Public duration transfer*

If long securities are retired and bills replace them, public duration falls. Treasury's refinancing exposure rises because the replacement reprices sooner. If securities of similar duration replace those retired, the duration effect is small and the case rests mainly on liquidity or cash management. If the purchase is funded from a fiscal surplus, the operation can reduce debt, but the surplus - not the buyback mechanism - created the solvency change.

### The September Observation

On September 9 the U.S. Treasury announced a maximum $6 billion long-end buyback for September 10. Reporting after the operation indicated that approximately $5.2 billion was accepted. Long Treasury yields nevertheless rose during a session also dominated by a sharp oil move, inflation data, and a thirty-year auction. [2] [3] The result is consistent with the proposition that a small liquidity operation cannot dominate a global repricing. It does not identify the counterfactual. Yields might have risen further without the operation; the operation may have improved relative-value liquidity even while the market level sold off.

Accordingly, this paper does not use the event as causal proof. The defensible test is narrower: measure liquidity metrics around eligible and ineligible securities, estimate changes in public DV01 and RaR after replacement financing, and test whether the fiscal convergence residual changed. If Γ did not change, the operation was not a fiscal adjustment, whatever its value as market maintenance.

## VI The Term Premium and the Demand Wedge

A long yield contains more than expected short rates. A useful decomposition writes an n-period yield as the average expected path of short rates plus a term premium. The premium compensates for duration risk, inflation uncertainty, balance-sheet scarcity, model risk, and the covariance of the bond with investors' marginal value of wealth. It is not observed directly. New York Fed estimates are model outputs and can be revised; they should be treated as a diagnostic range rather than as a measured price. [4]

$$y^{(n)}{}_t = (1 / n) \Sigma E_t i_{t+j} + TP^{(n)}{}_t$$

*Equation 12 Long-yield decomposition*

Debt management can affect the term premium by changing the quantity and duration of risk the public must absorb, but the sign is state-dependent. Retiring scarce off-the-run bonds can improve liquidity. Retiring duration while issuing bills can reduce duration supply but increase future rollover exposure. Announcing an operation in response to rising yields can also change beliefs about the fiscal authority's objective. Quantity, composition, and signalling must therefore be estimated separately.

### The Japanese Investor Decision

For a yen-based institution, the relevant comparison is not the unhedged U.S. yield against the JGB yield. It is the expected yen return after the hedge, cross-currency basis, transaction costs, capital treatment, and liability-matching value. A schematic one-period comparison is sufficient to show the ambiguity.

$$R^{UST \to JPY} \approx y^{US} - c^{hedge} - c^{basis} - c^{balance}$$

*Equation 13 Yen-hedged Treasury return*

$$\text{Demand wedge } \omega = R^{UST \to JPY} - y^{JP} - \text{liquidity and duration adjustments}$$

*Equation 14 Relative demand wedge*

A Bank of Japan rate increase can narrow the short-rate differential and reduce the mechanical cost of hedging dollars into yen, which supports hedged Treasury demand all else equal. The same increase can raise JGB yields, making the domestic alternative more attractive, which weakens Treasury demand all else equal. It can also strengthen the yen, alter the value of unhedged positions, and raise Japan's own debt-service path as liabilities roll. The net sign cannot be asserted from the policy-rate move alone. It depends on hedge ratios, basis, capital constraints, duration preference, and expected currency behaviour.

| Japanese rate-rise channel | First-order effect | Effect on hedged U.S. Treasury demand | Measurement |
|---|---|---|---|
| Narrower U.S.-Japan short-rate gap | Lower forward hedge cost, all else equal | Positive | Forward points and hedge tenor |
| Higher JGB yield | More competitive domestic asset | Negative | Hedged spread to matched-duration JGB |

| Japanese rate-rise channel | First-order effect | Effect on hedged U.S. Treasury demand | Measurement |
|---|---|---|---|
| Cross-currency basis | May amplify or offset covered-rate parity | State dependent | Basis by tenor |
| Stronger yen | Changes unhedged return and hedge demand | Investor dependent | Hedge ratio and FX expectation |
| Higher Japanese fiscal pass-through | Raises future JGB supply or consolidation pressure | Indirect and lagged | Japan q, RaR, debt-service budget |

This revises a simplified statement in Paper III that a Japanese rate rise necessarily thins the Treasury bid. It may do so through the domestic-yield channel; it may do the opposite through the hedge-cost channel. The institutional conclusion survives in a more rigorous form: Washington cannot infer foreign duration demand from the direction of another central bank's policy rate. It must observe the hedged demand wedge.

## VII The Coupled Washington Tokyo System

The two sovereign systems share investors, dealer balance sheets, currency hedges, inflation news, commodity shocks, and issuance calendars. Their debt ratios are not one equation, but the markets that price them are coupled. Define a local state vector containing U.S. and Japanese debt ratios, portfolio effective rates, term premia, the dollar-yen exchange rate, the cross-currency basis, and selected liquidity measures. Define a control vector containing fiscal measures, issuance composition, buybacks, policy rates, liquidity operations, and any authorised foreign-exchange intervention. Around a reference path, the system can be written in state-space form.

$$x_{t+1} - x^* = A(x_t - x^*) + Bu_t + \varepsilon_t$$

*Equation 15 Open-loop local system*

$$u_t = -K(x_t - x^*)$$

*Equation 16 State-contingent policy reaction*

$$A^{cl} = A - BK \quad \text{and} \quad \text{stability requires } \rho(A^{cl}) < 1$$

*Equation 17 Closed-loop convergence condition*

No numerical estimate of A, B, or K is claimed here. Equation 17 is an architecture. It says that the stability of the joint system depends not only on the underlying market transmission A, but also on which authority reacts to which variable and with what strength. Two individually rational controllers can destabilise the combined system if they push the same state in opposite directions or use one instrument to pursue incompatible targets.

### Local Optimisation and Global Instability

Suppose Washington responds to long yields with buybacks financed at shorter maturities while Tokyo responds to currency weakness with a policy-rate increase. Washington may reduce public duration today but raise U.S. q. Tokyo may reduce the hedge-cost differential but raise the JGB alternative and Japan's future effective rate. Each controller can meet a local objective while increasing the sensitivity of another state. The problem is not ill will. It is an uncoordinated gain matrix.

A credible Washington-Tokyo structure therefore does not require a promise to hold an exchange rate or a bond yield at a political number. It requires a shared map of transmission, explicit separation of domestic mandates, and operational readiness for the narrow market failures that each authority is legally equipped to address. Coordination is the publication of interfaces, not the merger of institutions.

## VIII Correct Instrument Assignment

The first discipline of control engineering is to avoid using the nearest actuator for every error signal. The target-instrument map below separates solvency, liquidity, refinancing, funding stress, exchange-rate disorder, and inflation. It also corrects Paper III's assignment of central-bank swap lines. The Federal Reserve describes its standing swap lines as facilities for improving liquidity in U.S. and foreign money markets during stress and for enabling foreign central banks to deliver dollar funding. [5] They are not a standing instrument for selecting a persistent spot exchange rate.

| Target | Observable | Proper actuator | Category error |
|---|---|---|---|
| Debt-ratio convergence | Γ, debt path, primary balance | Legislated fiscal reaction rule | Treasury buyback alone |
| Market liquidity | Depth, bid-ask, fails, off-the-run dispersion | Scheduled buybacks, switches, dealer facilities | Permanent fiscal contraction |
| Refinancing risk | RaR, q, maturity concentrations | Issuance maturity and indexation policy | Spot FX management |
| Dollar-funding stress | Cross-currency basis, funding spreads, swap usage | Central-bank liquidity swaps and domestic liquidity tools | Policy-rate path chosen for FX |
| Disorderly FX market | Depth, gaps, realised volatility, order imbalance | Communication and legally authorised FX intervention | Liquidity swaps treated as a peg |
| Domestic inflation | Inflation, expectations, wages, activity | Independent monetary policy | Debt-management operation |
| Natural duration demand | Auction tails, hedged spread, liability demand | Security design, predictable calendars, market structure | Implicit price support |

Instrument separation does not deny interaction. A liquidity operation can affect yields; a fiscal rule can affect the term premium; monetary policy can affect the exchange rate. The point is that the instrument's release condition should be tied to the failure it is designed to repair. Otherwise, the public cannot know whether an operation is temporary market maintenance or a substitute for adjustment. Ambiguity raises the very premium the operation is meant to compress.

## IX The Sovereign Convergence Rule

A credible fiscal package should be represented as a reaction function rather than as one heroic dollar announcement. The package must adapt when growth, the effective rate, or the debt stock differs from forecast. It must also distinguish emergency escape clauses from permanent abandonment. This paper calls that structure the Sovereign Convergence Rule.

$$b_t = \bar{b} - \varphi^d(d_{t-1} - \bar{d}) - \varphi^r \max[0, \bar{r}_t - g_t - \bar{m}]$$

*Equation 18  Illustrative sovereign fiscal reaction*

The symbols are deliberately generic. The reference primary deficit b-bar, debt anchor d-bar, and permitted interest-growth margin m-bar are political choices. The coefficients φd and φr govern how strongly the primary balance responds when debt or the effective-rate gap moves adversely. A rule is stabilising only if the resulting closed-loop dynamics place the dominant eigenvalue inside the unit circle under published stress scenarios. That must be tested, not asserted.

The Sovereign Convergence Rule has six governance requirements.

- Legislation specifies the multi-year primary-balance path and the measures that deliver it, rather than announcing an aspiration.
- An automatic correction mechanism responds to forecast error in debt, growth, and the effective rate within explicit bounds.

- Escape clauses are limited to named emergencies, identify the authority that activates them, and include a dated restoration path.
- Debt management publishes q and RaR alongside average maturity so that near-term rate pass-through is visible.
- Temporary liquidity operations publish eligibility, maximum size, observed market defect, and termination condition.
- An independent fiscal institution reports Γ under baseline, adverse, and severe-but-plausible states using common definitions.

The rule does not prescribe who pays. A revenue measure and an expenditure measure of equal budget value enter b identically but can have different growth, distributional, and political effects. Those second-round effects feed back into g and therefore into λ. The engineering contribution is to require the complete mapping and prevent a measure from being counted at face value while its growth effect is ignored.

### The GFE Fiscal State Vector

For public monitoring, the doctrine reduces the system to three headline quantities reported together.

| Quantity | Definition | Interpretation | Publication frequency |
|---|---|---|---|
| Fiscal eigenvalue λ | (1 + effective r) / (1 + nominal g) | Persistence of inherited debt under the current macro rate pair | Quarterly with revisions |
| Convergence residual Γ | Exact one-period debt drift | Positive is divergence; zero is stabilisation; negative is reduction | Quarterly and annual budget |
| Refinancing memory q | Share of debt repriced in the measurement interval | Speed at which marginal yields enter the fiscal account | Monthly profile and annual summary |

No one element is sufficient. λ without Γ can call a high-debt fixed point stable. Γ without q can understate how quickly today's rate shock reaches tomorrow's budget. q without λ can mistake slow pass-through for solvency. The state vector makes the interaction explicit.

## X Stress Testing and Falsification

An advanced doctrine must expose the observations that could prove it wrong. This framework is not falsified because a single auction is strong or a single buyback day is weak. It is falsified if its stated transmission does not survive repeated measurement after confounding factors are controlled.

| Hypothesis | Test | Evidence against the framework |
|---|---|---|
| Refinancing memory governs pass-through | Apply the published maturity kernel to past yield cohorts and compare with realised effective rates | Systematic forecast failure unrelated to composition or stock-flow adjustments |
| Fiscal forcing matters for the long end | Estimate term-premium response to credible legislative fiscal news, controlling for inflation and policy expectations | No persistent response across identified episodes |
| Buybacks are chiefly liquidity and composition tools | Compare eligible and matched ineligible securities plus post-financing DV01 and RaR | Large durable Γ improvement without accompanying primary or stock-flow change |
| Japanese rate effects are channel-ambiguous | Decompose hedged spread, domestic alternative, basis, and investor flows | One invariant sign across states with no role for hedge cost or JGB yield |

| Hypothesis | Test | Evidence against the framework |
|---|---|---|
| Instrument assignment lowers control conflict | Compare market response when release conditions are explicit against ambiguous interventions | No difference after accounting for size and state |

### Published Monitoring Grid

The minimum public grid should include the following series and vintages. Revisions must be retained because a rule evaluated only on revised data can appear more precise than the information available to the decision maker in real time.

- Fiscal: primary balance, net interest, debt held by the public, stock-flow adjustments, and the vintage of nominal GDP.
- Debt management: maturity buckets, bill share, floating and inflation-linked exposure, weighted average maturity, q, RaR, net DV01 issued, and buyback results.
- Market: expected-rate path, multiple term-premium estimates, auction tails, dealer inventories, fails, depth, and bid-ask dispersion.
- Cross-border: Treasury holdings by residence with reporting lags, matched-duration hedged spreads, forward points, cross-currency basis, and investor-class flow data where available.
- Japan: effective government rate, maturity profile, debt-service budget, JGB ownership, Bank of Japan holdings, JGB yields, and the fiscal assumptions applied to budget requests.

Term-premium models disagree and holdings data arrive with lags. Those are measurement limits, not reasons to omit the variables. The proper response is to publish ranges, revisions, and confidence grades. False precision is a larger governance failure than acknowledged uncertainty.

## XI The Executed Institutional Argument

The public doctrine series reports the institution's own book retrospectively when that book supplies evidence. Paper III recorded that GATS7 held short Euro-Bund futures across sixteen legs and long U.S. and European equity-index futures while holding no position in the U.S. Treasury note or bond futures and no Japanese-yen futures. This companion makes no new position claim and offers no trade expression. The instruments at the centre of its model remained outside the cited book. The evidence here is the identity, the official fiscal series, the maturity logic, and the observable post-operation record.

### Doctrine Restated

The institutional law remains: hold the machine still and cut only on a named defect. The technical advance is that the machine now has a declared state vector. A controller that observes only the market yield may re-tune the buyback size indefinitely. A controller that observes $\lambda$, $\Gamma$, and q can distinguish a liquidity defect from a fiscal forcing error and a fast-refinancing defect. The state determines the intervention.

### Consequence

The September operation supplies one lesson of classification. Raising a buyback ceiling can be rational market maintenance. It becomes dangerous doctrine only when its headline size is allowed to stand in for a change in $\Gamma$. The correction in this paper supplies a second lesson. When an official direct series exists, a rounded derived input must yield to it. The institution's credibility does not lie in never correcting a number. It lies in registering the correction, recomputing the result, and exposing the new boundary.

# XII Conclusion

The arithmetic of escape begins with a deficit but ends with a stability condition. The primary deficit is the forcing term. The fiscal eigenvalue transmits the inherited stock. Refinancing memory determines when market rates enter the budget. A sovereign escapes only when the combined convergence residual is brought to zero or below and the policy reaction keeps the dominant closed-loop eigenvalue inside the unit circle across credible shocks.

This framework changes the interpretation of buybacks. They can be valuable for liquidity, cash management, and duration composition. They cannot, without accompanying fiscal resources, remove the primary forcing term. Their correct evaluation is therefore not did yields fall on the day, but which ledger changed, by how much, for how long, and with what replacement financing.

It also changes the Washington-Tokyo argument. A Japanese policy-rate rise does not have one mechanical effect on Treasury demand. It lowers one hedge-cost wedge, raises one domestic-yield alternative, changes the currency state, and enters Japan's own interest bill with a refinancing lag. The net result is an empirical balance of channels. Swap lines belong to funding stress. Foreign-exchange tools belong to disorderly currency markets. Monetary policy belongs to domestic price stability. Fiscal convergence belongs to legislated primary policy.

The advanced title is therefore literal. The fiscal eigenvalue is the number that tells an institution whether inherited debt disturbances decay. It is not the number that tells it where debt will settle. For that, the forcing term and the refinancing kernel must be restored. A system can converge and still have no acceptable destination. The engineering task is not merely to lower the gain. It is to change the forcing, lengthen the memory where insurance is worth its price, and assign every actuator to the defect it can actually repair.

> *A sovereign has not escaped when the eigenvalue falls below one. It has escaped when the destination is finite, the residual is non-positive, and the rule survives the next shock.*

### Sequence Note

Paper IV of The Engineered Long End remains Yields Rising with a Falling Dollar  Reading a Term Premium Regime from a Book That Has No Opinion.

# Appendix A Derivations

### From Nominal Debt to the Debt Ratio

Begin with $D_t = (1 + r_t)D_{t-1} + P_t + F_t$ and $Y_t = (1 + g_t)Y_{t-1}$. Divide the first identity by $Y_t$. Because $D_{t-1} / Y_t$ equals $d_{t-1} / (1 + g_t)$, the result is Equation 1. Subtract $d_{t-1}$ from both sides and combine the coefficient terms to obtain Equation 2. Set Δd equal to zero and solve for b to obtain Equation 6. Hold b, r, g, and s constant and solve $d = \lambda d + b + s$ to obtain Equation 4.

### Why the Eigenvalue Is Necessary but Not Sufficient

For the scalar system $d_t = \lambda d_{t-1} + f$, where $f = b + s$, the solution after T periods is $d_{t+}{}^{T} = \lambda^{T} d_t + f(1 - \lambda^{T}) / (1 - \lambda)$ when λ is not one. If the absolute value of λ is below one, the initial-condition term vanishes as T grows, but the forcing term converges to $f / (1 - \lambda)$. When λ is close to one, that quotient can be extremely large. This is the mathematical distinction between convergence and an acceptable fixed point.

$$d_{t+}{}^{T} = \lambda^{T}\, d_t + f(1 - \lambda^{T}) \;/\; (1 - \lambda)$$

*Equation A1  Scalar debt solution with constant forcing*

### Rate Shock with Refinancing Memory

If the marginal refinancing yield rises permanently by Δy and q is constant, the effective-rate change after T periods is Δr-bar equal to Δy times [1 - (1 - q) to the power T]. The first-year budget effect is therefore approximately qdΔy rather than dΔy. The full-stock sensitivity dΔy is the limiting effect after the old portfolio has rolled, subject to changes in debt, growth, and composition.

$$\Delta\bar{r}_{t+}{}^{T} = \Delta y\; [1 - (1 - q)^{T}]$$

*Equation A2  Pass-through of a permanent marginal-rate shock*

$$\text{First-period } \Delta\text{interest} \;/\; Y \approx q\; d\; \Delta y$$

*Equation A3  Near-term budget sensitivity*

Actual sovereign portfolios require cohort-level cash flows, separate treatment of bills, notes, bonds, floating-rate notes, and inflation-protected securities, plus expected future deficits. The renewal equation is a transparent approximation, not a substitute for the official debt-service model.

# Appendix B Source and Method Note

This paper uses official sources for fiscal identities, baseline projections, debt-management descriptions, term-premium diagnostics, and liquidity-swap design. Current-event reporting is used only for the announced and accepted buyback sizes and contemporaneous market context. Calculations are the authors' arithmetic from rounded published inputs and may not reproduce unrounded agency work. Monetary and fiscal variables use nominal rates unless stated otherwise. Debt means U.S. federal debt held by the public when calibrating the U.S. equation; gross general-government measures should not be substituted into that calibration without reconciling coverage.

The U.S.-Japan state-space model is conceptual. It identifies channels and stability conditions but does not estimate a structural matrix or claim a causal coefficient. Signs described as ambiguous are intentionally left for measurement. Term premiums are latent and model-dependent. Cross-border holdings data are reported by residence and may not identify ultimate beneficial owners.

### Sources

**[1] Congressional Budget Office.** *The Budget and Economic Outlook  2026 to 2036. February 2026.* https://www.cbo.gov/publication/62105 Baseline debt, primary deficit, net interest, average effective rate, and maturity-based interest projections.

[2] **Reuters.** *U.S. Treasury to buy up to $6 billion in September 10 buyback operation. 9 September 2026.* https://www.reuters.com/world/us-treasury-buy-up-6-billion-sept-10-buyback-operation-2026-09-09/ Announcement and market context.

[3] **Financial Times.** *Global bond sell-off reignites as oil jumps to $109. 10 September 2026.* https://www.ft.com/content/49e8c693-0064-4ef9-917f-5f2be97458fc Reported acceptance of approximately $5.2 billion and contemporaneous yield context.

[4] **Federal Reserve Bank of New York.** *Treasury Term Premia. accessed 10 September 2026.* https://www.newyorkfed.org/research/data_indicators/term-premia-tabs Model-based term-premium estimates and methodological links.

[5] **Federal Reserve Bank of New York.** *Central Bank Swap Arrangements. accessed 10 September 2026.* https://www.newyorkfed.org/markets/international-market-operations/central-bank-swap-arrangements Standing liquidity-swap purpose and counterparties, including the Bank of Japan.

[6] **Ministry of Finance Japan.** *Debt Management Report 2025. 2025.* https://www.mof.go.jp/english/policy/jgbs/publication/debt_management_report/2025/index.html JGB issuance, maturity, risk-management, and holder context.

[7] **Reuters.** *Japan budget requests swell to pandemic-era scale under Takaichi agenda. 4 September 2026.* https://www.reuters.com/world/asia-pacific/japan-budget-requests-swell-pandemic-era-scale-under-takaichi-agenda-2026-09-04/ Current budget-request and debt-service assumptions.

[8] **Reuters.** *BOJ to raise rates to 1.25 percent this month reach 1.75 percent faster than expected. 9 September 2026.* https://www.reuters.com/world/asia-pacific/boj-raise-rates-125-this-month-reach-175-faster-than-expected-2026-09-09/ Contemporaneous survey context; not used as an institutional forecast.

[9] **U.S. Department of the Treasury.** *Minutes of the Meeting of the Treasury Borrowing Advisory Committee November 2 2021. 2 November 2021.* https://home.treasury.gov/news/press-releases/jy0464 Historical advisory discussion of a medium-term bill-share range and debt-management flexibility.

## Selected Analytical Lineage

The exact debt identity is standard public-finance arithmetic. The distinction between debt dynamics and welfare follows the literature on interest-growth differentials; the separation of expected short rates from a term premium follows affine term-structure work; and the treatment of maturity as a cost-risk choice follows the sovereign debt-management literature. The terms Fiscal Eigenvalue, Convergence Residual, Refinancing Memory, GFE Fiscal State Vector, and Sovereign Convergence Rule are used here as GFE doctrine to organise those established mechanics into one operational control framework.

[10] **Olivier Blanchard American Economic Review.** *Public Debt and Low Interest Rates. 2019.* https://doi.org/10.1257/aer.109.4.1197 Interest-growth differentials, debt dynamics, and welfare distinctions.

[11] **Tobias Adrian Richard Crump and Emanuel Moench Journal of Financial Economics.** *Pricing the Term Structure with Linear Regressions. 2013.* https://doi.org/10.1016/j.jfineco.2013.04.009 Term-structure decomposition and model-based term premium.

[12] **Robin Greenwood Samuel Hanson and Jeremy Stein Journal of Finance.** *A Comparative Advantage Approach to Government Debt Maturity. 2015.* https://doi.org/10.1111/jofi.12253 Government debt maturity and the supply of duration risk.

# Institutional Disclosures

## About the Author

Dr. Glen Brown is President and CEO of Global Financial Engineering Inc and Global Accountancy Institute Inc. He is the architect of the GCPIAUT framework and the broader GATS-native proprietary capital doctrine. His work focuses on sovereign capital governance, algorithmic trading architecture, reserve-first compounding systems, and institutional financial engineering within closed proprietary environments.

## Business Model Clarification

Global Financial Engineering Inc and Global Accountancy Institute Inc operate as closed-loop proprietary institutions. They do not offer public investment products through the doctrines described here, do not invite retail participation into their proprietary capital architecture, and do not present the GCPIAUT framework as a public collective investment scheme. The concepts discussed in this paper form part of the firms' internal intellectual and operating doctrine.

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